# Anomalous Behavior of the Spin Susceptibility of Strongly Correlated Fermi Systems

J. W. Clark,[1] V. A. Khodel,[2,1] and M. V. Zverev[2]

[1] *McDonnell Center for the Space Sciences and Department of Physics, Washington University, St. Louis, MO 63130, USA*

[2] *Russian Research Centre Kurchatov Institute, Moscow, 123182, Russia*

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The spin susceptibility $\chi(T)$ of strongly correlated Fermi systems is investigated in the density region where Fermi-liquid theory fails. We attribute this failure to a specific quantum phase transition associated with a rearrangement of the Landau state at low temperatures $T$, retaining the assumption that the Landau quasiparticle picture survives in a generic sense. Taking into account the resulting modification of the quasiparticle distribution function, the spin susceptibility $\chi(T)$ is shown to contain a Curie-Weiss component $\chi_{\rm CW}(T) \sim (T-\Theta_{\rm W})^{-1}$, with the Weiss temperature $\Theta_{\rm W}$ vanishing at the critical density for the transition.

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The manifestation of non-Fermi-liquid behavior in strongly correlated Fermi systems provides valuable clues to a fundamental microscopic understanding of these systems. This letter was stimulated by the findings of recent studies of laboratory simulants of two-dimensional (2D) liquid $^3$He and the electron gas.[1,2,3,4,5,6,7,8] Two dramatic effects have been revealed by data for the spin susceptibility $\chi(T)$ and specific heat of 2D liquid $^3$He and for the temperature dependence of Shubnikov-de Haas conductivity oscillations in the 2D electron gas. First, the results point to a divergence of the effective mass $M^*$ in these systems at a critical density $\rho_\infty$ where, concomitantly, the single-particle spectrum $\varepsilon(p)$ becomes flat. Second, as seen in the experimental data[1,2,5] on $\chi(T\to 0)$, Landau theory begins to fail even before this critical density is reached.

Ordinarily, the non-Fermi-liquid behavior of $\chi(T\to 0)$ is attributed to a disorder-driven localization transition. To be sure, disorder plays a part, as evidenced in data[9] on the spin susceptibility of submonolayer $^3$He at density $\rho = 0.0064\,\text{Å}^{-2}(\sim 0.1$ layer), absorbed onto a Nuclepore substrate pre-plated with several $^4$He layers. The spin susceptibility of this low-density disordered system is found to contain a significant Curie component $\chi_C(T) \sim T^{-1}$, which progressively diminishes with increase of $^4$He coverage $D_4$, vanishing at $D_4 \simeq 2.5$ layers. However, such a Curie component $\chi_C(T)$ is also found in the spin susceptibility of a high-mobility 2D layer in silicon.[5] In this case, disorder effects are suppressed, while the effective interaction between electrons is strong, since the $r_s$ parameter attains values around 10. Furthermore, the same behavior of $\chi(T\to 0)$ is observed[1,2] when two monolayers of $^3$He are absorbed onto a graphite substrate pre-plated with a monolayer of $^4$He. The density of liquid $^3$He in this experiment is about an order of magnitude higher than in the work of Ref. 9. Again the effective interaction is strong, since $M^*$ is drastically enhanced. Thus, at least two examples exist in which the interaction appears to be the dominant playmaker, rather than disorder.

If indeed disorder proves to be irrelevant in some strongly interacting Fermi systems, how does the interaction give rise the inferred singular behavior of $\chi(T\to 0)$? A possible answer to this question is provided by a microscopic mechanism proposed over a decade ago,[10] which predicted a flattening of the single-particle spectrum in strongly correlated Fermi systems well before the phenomenon was observed in angle-resolved photoemission studies.[11,12] In this scenario, known as fermion condensation,[13,14,15,16,17,18,19] the flattening effect is associated with a phase transition in which the conventional Landau state suffers a rearrangement. It is intrinsic to this model that the Landau-Migdal quasiparticle picture retains its validity beyond the critical point. However, at $T=0$ the Fermi step $n_F(p)=\theta(p_F-p)$ is replaced by a new momentum distribution, $n_0(p)$, determined by the variational condition

$$\delta E/\delta n(p) = \mu \ . \tag{1}$$

The distribution $n_0(p)$ differs from $n_F(p)$ in a finite momentum interval $p_i<p<p_f$ that includes the Fermi momentum $p_F$. Since the variational derivative $\delta E/\delta n(p)$ is by definition the energy $\epsilon(p)$ of a Landau quasiparticle, the condition (1) implies that in this domain, the single-particle spectrum must be completely flat, pinned to the chemical potential $\mu$:

$$\epsilon(p)=\mu\ , \quad p_i<p<p_f\ . \tag{2}$$

The set of single-particle states having $\varepsilon(p)=\mu$ is called the fermion condensate (FC) in analogy with the Bose condensate existing in liquid $^4$He, since both condensates entail similar $\delta$-like singular contributions to the density of states.

At finite $T$, the degeneracy of the FC spectrum is lifted in accordance with the formula[14]

$$\xi(p,T\to 0)\equiv \epsilon(p)-\mu = T\ln\frac{1-n_0(p)}{n_0(p)}\ , \quad p_i<p<p_f\ , \tag{3}$$

which ensures consistency between the Fermi-Dirac expression $n(p,T)=[1+\exp(\xi(p)/T)]^{-1}$ and the solution

$n_0(p)$ of Eq. (1) at $T=0$. At higher $T$ the FC density falls to disappear at the critical temperature $T_f$.

Having set the stage, let us now evaluate the spin susceptibility $\chi$ of a strongly correlated Fermi system that experiences a rearrangement of the Landau state at critical temperature $T_f$. It is instructive to begin on the "metallic" side of the transition, where standard Landau theory still holds. Within the quasiparticle picture,[20]

$$\chi = \frac{\chi_0}{1 - g_0\Pi_0} \ , \qquad (4)$$

where $g_0$ is the zeroth harmonic of the Landau spin-spin interaction and $\chi_0 = -\mu_B^2\Pi_0$ is the spin susceptibility of the noninteracting quasiparticles in terms of the Bohr magneton $\mu_B$. The corresponding polarization operator is

$$\Pi_0(T) = \int \frac{dn(\xi(p))}{d\xi(p)} d\tau \equiv v_F^0 P_L I(T) \ , \qquad (5)$$

where $v_F^0 = \partial\varepsilon_p^0/\partial p$ is the quasiparticle group velocity and $P_L$ is the conventional $T$-independent, $D$-dimensional Landau result for $\Pi_0(T)$, while

$$I(T) = -\int_{-\infty}^{\infty} \frac{dn(\xi,T)}{d\xi}\frac{dp(\xi)}{d\xi} d\xi \ . \qquad (6)$$

The derivation of Eq. (4) takes account of the fact that the renormalization factor $z$ specifying the quasiparticle weight enters only in the phenomenological parameter $g_0$. According to Migdal's relation $\mathcal{T}(\boldsymbol{\sigma}; k=0, \omega\to 0) = \partial G^{-1}(\varepsilon)/\partial\varepsilon \equiv z^{-1}$ stemming from spin conservation, effects of renomalization of the vertex parts $\mathcal{T}$ and the Green functions $G$ cancel.

Noting that $dn(\xi,T)/d\xi$ decays rapidly at $\xi \geq T$, Rolle's theorem may be invoked to reduce integral (6) to the value of the function $dp(\xi)/d\xi$ at a certain point $\xi_R \sim T$. In strongly correlated Fermi systems that exhibit non-Fermi-liquid behavior, there are two distinct temperature-density regions in which the function $(dp(\xi)/d\xi)_{\xi_R}$ shows qualitatively different departures from the Landau norm. The first region is situated close to the critical density $\rho_\infty$. A portion of spectrum $\xi(p)$ adjacent to the Fermi surface is flattened anomalously, and the curve $dp(\xi)/d\xi$ bends downward, producing $T$-dependent corrections to $I(T)$, and hence to $\chi(T)$, that become important at sufficiently high $T$. In 2D liquid $^3$He, this behavior is successfully described by the phenomenological formula[21] $\chi(T) \sim \left[(T^{**}(\rho))^2 + T^2\right]^{-1/2}$. Still within the first region, Landau theory fails at low $T \sim 0$ as well. New terms in the Taylor expansion,

$$\xi(p,\rho) = p_F\frac{p-p_F}{M^*(\rho)} + \xi_3\frac{(p-p_F)^3}{p_F^3} + \cdots \ , \qquad (7)$$

odd in $p - p_F$, come into play since the first term is suppressed by the divergence of the effective mass. Neglecting the $M^*$ term, the cubic term becomes dominant and upon inserting Eq. (7) into Eq. (5) one finds

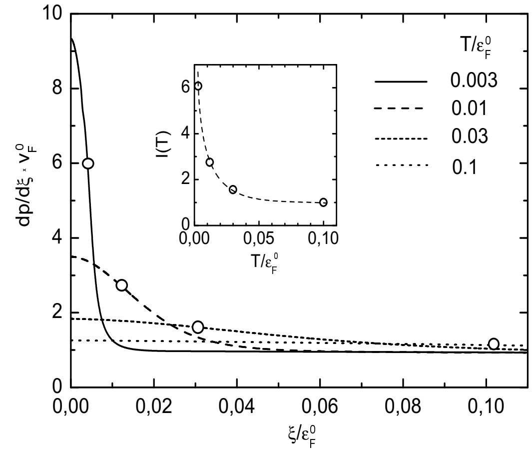


FIG. 1: Derivative $dp(\xi)/d\xi$, in units of $(v_F^0)^{-1}$, plotted as a function of $\xi/\varepsilon_F^0$, based on calculations in the Nozières model at different temperatures $T$. Open circles label the points ($\xi_R$, $dp/d\xi|_R$) on the corresponding curves. Inset: The integral $I(T)$, evaluated as $(dp/d\xi)_{\xi_R}$, is drawn versus $T/\varepsilon_F^0$.

$\Pi_0(T\to 0) \sim -T^{-2/3}$. In the second region, $T \ll T_f(\rho)$ and the FC has come to occupy a significant fraction of the Fermi sphere. In this case, substituting Eq. (3) into Eq. (5) yields $\Pi_0(T\to 0) \sim -1/T$.

The foregoing inferences are confirmed in numerical calculations. For example, Fig. 1 shows results for the functions $dp/d\xi$ and $I(T)$ based on the Nozières model,[14] in which the effective interaction between particles is proportional to $\delta(\mathbf{p}_1 - \mathbf{p}_2)$. The transitional behavior near $\rho_\infty$ is absent in this model, and the function $I(T)$ is simply the sum of a term $\sim 1/T$ and a constant term.

In arriving at these results, we have neglected damping, which is more pronounced in systems containing a FC than in ordinary Fermi liquids. However, in the energy region $\varepsilon \simeq T$ relevant to the current problem, the damping parameter is moderate, with $\gamma(T) \sim T$. As a consequence, damping does not affect the predicted form of $\Pi_0(T\to 0)$, instead altering only numerical factors.[19,22] In the transitional region where $\xi(p)$ is not completely flat, the ratio $\gamma(T)/T$ is below 1; hence the inclusion of damping has little effect.

We conclude that there exist two regimes in which the spin susceptibility shows characteristic variation with temperature, in addition to the standard Fermi-liquid domain. The first is a transitional region close to the point of fermion condensation, where the FC is incipient or still minute. Upon inserting the above result $\Pi_0(T\to 0) \sim -T^{-2/3}$ into the Eq. (4), simple algebra leads to

$$\chi(T\to 0) \sim (C_t T^{2/3} - \Theta_t)^{-1} \ , \qquad (8)$$

where $\Theta_t \sim -g_0\rho$. Such a power-law $T$-dependence is observed in a number of heavy-fermion metals.[23,24]

As the density increases, we pass from the transitional regime characterized by (8) to the region where the FC

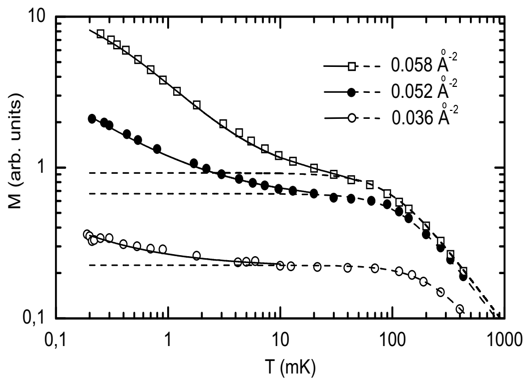


FIG. 2: Magnetization for $^3$He films at different densities. Experimental data from Refs. 1,2 are indicated by symbols, solid curves show the present predictions at low $T$, and dashed lines indicate the phenomenological fit of Ref. 21 at higher $T$.

contribution plays the dominant role. On combining Eqs. (4), (5), and the result $\Pi_0(T \to 0) \sim -1/T$, we obtain the Curie-Weiss formula

$$\chi(T \to 0) \sim (T - \Theta_W)^{-1} \ , \tag{9}$$

with a Weiss temperature $\Theta_W \sim -g_0(\rho)\rho$. Remarkably, working within the general framework of the Landau quasiparticle approach, we have found conditions under which a strongly correlated Fermi liquid behaves as a gas of localized spins.

As a rule, the value of the Weiss temperature $\Theta_W$ is small (e.g., $\Theta_W \simeq -0.3$ mK for the uppermost curve in Fig. 2). The reason for this is that there is no ferromagnetic instability in any of the systems under consideration, i.e. the Pomeranchuck stability condition $1+g_0N(0) > 0$ is not violated. Since the density of states $N(0)$ at the Fermi surface is proportional to $M^*$, preservation of this condition implies that $g_0(\rho)$ changes its sign at the critical density $\rho_\infty$ where $M^* \to \infty$. Hence, at this density the character of the correlations changes from ferromagnetic to antiferromagnetic; such a change does take place in strongly correlated 2D liquid $^3$He (see Fig. 1 of Ref. 1).

The predicted behaviors (8) and (9) explain the measurements that trace the gradual evolution of the spin susceptibility of a 2D electron layer in silicon as the density $\rho$ is lowered.[5] The constant susceptibility of the ordinary Fermi liquid gives way to the $T$-dependent behavior $\chi(T) \sim T^{-1}$ of a gas of localized spins. The same trends are seen in the extensive data on the magnetization of $^3$He films reported in Refs. 1,2, some of which are plotted in Fig. 2 along with our theoretical predictions.

Before focusing on these results, it must be remarked that comparison of different sets of data on 2D liquid $^3$He reveals a crucial dependence of the effective-mass enhancement on the number of $^4$He layers separating the $^3$He monolayer from the graphite substrate[25]: The smaller the number, the greater is the increase of $M^*$. It would appear that the enhancement of $M^*$ is somehow associated with the crystal lattice of the substrate. A similar situation occurs in the flattening of single-particle spectra of 2D superconductors. For example, in the case of a quadratic lattice, flattening is most evident in the region adjacent to the van Hove points.[11,12] As seen in ARPES data[26] on $Na_{0.5}VO_2$, flat portions of the single-particle spectrum exist in 2D metals having the same Brillouin zone as 2D liquid $^3$He. Therefore we suggest that the formation of FC in certain segments of the Fermi line of 2D liquid $^3$He and, correspondingly, the emergence of Curie-Weiss components in its spin susceptibility, begins at densities markedly lower than the threshold for fermion condensation in homogeneous 2D liquid $^3$He.

Turning now to Fig. 2, we first consider the uppermost curve, belonging to the highest density $\rho = 0.058\,\text{Å}^{-2}$, where it is assumed that the FC occupies a significant part of the Fermi sphere. The corresponding contribution to the spin susceptibility $\chi(T \to 0)$ is evaluated by means of Eq. (9). To accomodate higher $T \sim T_f$, we suppose that upon approach to $T_f$ the FC density behaves as $(1 - T^2/T_f^2)^{1/2}$, with $T_f = 30\,\text{mK}$. (In fact, the specific form of this attenuation factor is immaterial.) The contribution to $\Pi_0(T)$ from the remaining, normal part of the Fermi liquid is assumed to be constant at a value $\simeq 0.92$. Given these two parameter choices, the experimental data for $\chi(T, \rho = 0.058\,\text{Å}^{-2})$ are well reproduced over more than two orders of magnitude on the mK scale.

The other two curves in Fig. 2, for the densities $\rho = 0.052\,\text{Å}^{-2}$ and $0.036\,\text{Å}^{-2}$, are determined under the assumption that FC is poised on the verge of onset, so that the cubic term prevails in the series representation (7) of the spectrum $\xi(p)$. As for the uppermost curve, the effect of flattening on $\chi(T \to 0)$ in this transition regime is attenuated with rising $T$ through a factor $(1 - T^2/T_t^2)^{1/2}$. The constant contribution to $\Pi_0(T)$ from other effects is taken as $\simeq 0.04$ for $\rho = 0.052\,\text{Å}^{-2}$, and $\simeq 0.014$ for $\rho = 0.036\,\text{Å}^{-2}$. The remaining parameter choices are: $T_t = 20\,\text{mK}$, $C_t = 1.2\,\text{mK}^{1/3}$ for $\rho = 0.052\,\text{Å}^{-2}$; $T_t = 10\,\text{mK}$, $C_t = 17\,\text{mK}^{1/3}$ for $\rho = 0.036\,\text{Å}^{-2}$; and $\Theta_t = 0$ at both densities.

When the temperature is lowered to values such that $T < T_Z = \mu_B B$, where $B$ is the strength of a magnetic field applied to the sample, the perturbative approach employed above in evaluating $\chi(T)$ becomes questionable. One then needs to solve equations for the spectra $\xi_\pm(p)$ directly. These equations may be written in the form

$$\begin{aligned} \xi_+(p) &= \xi_p^0 + \frac{1}{2}T_Z + \int f(\mathbf{p}, \mathbf{p}_1)\frac{1}{2}\left[n_+(p_1)+n_-(p_1)\right] d\tau_1 \ , \\ \xi_-(p) &= \xi_+(p) - T_Z \ , \end{aligned} \tag{10}$$

where $n_+$ and $n_-$ are the quasiparticle momentum distributions of the subsystems with spin projections $\pm\frac{1}{2}$. The interaction function $f$ is assumed to be independent of the momentum distributions $n_+$ and $n_-$ themselves.

If the field $B$ is small, the solution of Eqs. (10) can be

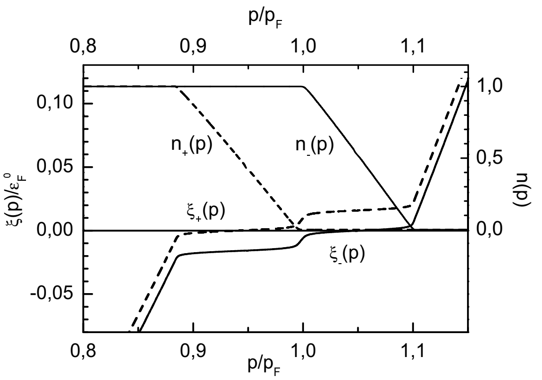


FIG. 3: Single-particle spectra $\xi_+(p)$ (dashed line) and $\xi_-(p)$ (solid line) in units of $\varepsilon_F^0$, together with momentum distributions $n_+(p)$ (dashed line) and $n_-(p)$ (solid line), plotted as functions of $p/p_F$ for $T_z/\varepsilon_F^0 = 10^{-2}$ and $T/\varepsilon_F^0 = 10^{-3}$. The Nozières model is assumed.

expressed in terms of the solution of the FC problem

$$\xi(p) = \xi_p^0 + \int f(\mathbf{p}, \mathbf{p}_1)\, n_0(p_1)\, d\tau_1 \qquad (11)$$

in the absence of the field. Setting $B = 0$ in Eqs. (10), we are led to the conclusion that the sum $[n_+(p) + n_-(p)]/2$ must coincide with the solution $n_0(p)$ of Eq. (11). But according to the second of Eqs. (10), only one of the two states with the same momentum $p$ can belong to the FC. In particular, if $n_0(p) > 1/2$, then $n_-(p) = 1$ and $n_+(p) = 2n_0(p) - 1$; hence, in the FC region one obtains $\xi_-(p) \simeq -T_z$, while $\xi_+(p) \simeq T\ln[(1-n_0(p))/(n_0(p)-1/2)]$. Conversely, if $n_0(p) < 1/2$, then $n_-(p) = 2n_0(p)$ and $n_+(p) = 0$; hence, in the FC region one has $\xi_-(p) \simeq T\ln[(1/2-n_0(p))/n_0(p)]$, while $\xi_+(p) \simeq T_Z$. Thus we infer that at the point where $n_0(p) = 1/2$, there is a jump in the single-particle energies. This behavior is illustrated in Fig. 3, where the particle energies $\xi_+(p)$ and $\xi_-(p)$ are plotted for the Nozières model with coupling constant $f/\varepsilon_F^0 = 0.2$.

Having obtained these results, the FC magnetization $\mathcal{M}^f$ is readily evaluated as sum of the integral over $n_0(p)$ from $n_0(p) = 0$ to $1/2$, and the integral of $1 - n_0(p)$ from $n_0(p) = 1/2$ to 1. Thus we see that in the case $\mu_B B > T_Z$, the FC magnetization $\mathcal{M}^f$ is independent of both $T$ and $B$. This property implies that the spin susceptibility, determined as the derivative $\chi = \partial\mathcal{M}/\partial B$, does not contain the FC contribution. We therefore conclude that at finite values of $B$ and low $T$, the spin susceptibility $\chi(T)$ of a system containing a FC increases with temperature until $T$ attains values comparable to $T_Z$, and then begins to fall off as $T^{-1}$ if the FC fraction is significant.

In summary, we have shown that if the interaction between particles is strong enough to induce a rearrangement of the Landau state, then the behavior of the spin susceptibility $\chi(T \to 0)$ progressively changes from that inherent in Landau Fermi-liquid theory to the behavior characteristic of a gas of localized spins. This finding calls for reexamination of many aspects of the theory of metallic oxides and heavy metals with localized spins.

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